The Distributional Dynamics of Income, Earnings and Consumption

JAE Lectures

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Setting the Scene

- Inequality has many linked dimensions: wages, incomes and consumption
- The link between the various types of inequality is mediated by multiple insurance mechanisms
- including labour supply, taxation, consumption smoothing, informal mechanisms, etc
- Wages► earnings► joint earnings► income► consumption
  - hours
    - Family labour supply
      - Taxes and transfers
        - Self-insurance/ partial-insurance/ advance information

‘Insurance’ mechanisms...

- These mechanisms will vary in importance across different types of households at different points of their life-cycle and at different points in time.
- The manner and scope for insurance depends on the durability of income shocks
- The objective here is to understand the distributional dynamics of wages, earnings, income and consumption
- That is to understand the transmission between wages, earnings, income and consumption inequality
  - **1980s in the US and UK have particularly interesting episodes, also Japan and Australia =$>$**

These lectures are an attempt to reconcile three key literatures:

► I. Examination of the evolution in inequality over time for consumption and income

● In particular, studies from the BLS, Johnson and Smeeding (2005); early work in the US by Cutler and Katz (1992) and in the UK by Blundell and Preston (1991) and Atkinson (1997), etc - **Table I**

These lectures are an attempt to reconcile three key literatures:

► I. Examination of inequality over time via consumption and income

► II. Econometric work on the panel data decomposition of the income process

● Lillard and Willis (1978), Lillard and Weiss (1979), MaCurdy(1982), Abowd and Card (1989), Gottschalk and Moffitt (1995, 2004), Baker (1997), Dickens (2000), Haider (2001), Meghir and Pistaferri (2004), Browning, Ejrnaes and Alverez (2007), Haider and Solon (2006), etc econometric work on the panel data decomposition of income processes

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- ► II. Econometric work on the panel data decomposition of the income process
- ► II. Work on intertemporal decisions under uncertainty, especially on partial insurance, excess sensitivity:

● Hall and Mishkin (1982), Campbell and Deaton (1989), Cochrane (1991), Deaton and Paxson (1994), Attanasio and Davis (1996), Blundell and Preston (1998), Krueger and Perri (2004, 2006), Heathcote et al (2005), Storresletten et al (2004), Attanasio and Pavoni (2006), Primiceri and Van Rens (2006), etc

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- ► III. Work on intertemporal decisions under uncertainty, especially on partial insurance, excess sensitivity:

• Hall and Mishkin (1982), Campbell and Deaton (1989), Cochrane (1991), Deaton and Paxson (1994), Attanasio and Davis (1996), Blundell and Preston (1998), Krueger and Perri (2006), Heathcote et al (2005), Storresletten et al (2004), Attanasio and Pavoni (2006), Primiceri and Van Rens (2006), Blundell, Pistaferri and Preston, etc

• Information and human capital:

• Cuhna, Heckman and Navarro (2005, 2007), Guvenen (2006, Huggett et al (2007)

JAE Lecture I:

- Distributional Dynamics of Income, Earnings and Consumption
- Developing the Transmission Parameter or ‘Partial Insurance’ approach:
  - What do we do?
  - What do we find?

JAE Lecture II:

- How well does the Partial Insurance approach work?
  - Robustness to alternative representations of the economy
  - Bewley economy, alternative economies - draw on simulation studies
- Are there other key avenues for ‘insurance’?
- What features need developing/generalising?

► Some resilient features of the distribution of consumption

- Construct quantile-quantile (QQ) plots as well as histograms of the sample.
- The QQ plot depicts the points $\{y_{(i)}, \mu + \sigma\Phi^{-1}(\frac{i}{n})\}$ for $i = 1, ..., n$.
- Use robust estimates for location and scale parameters μ and σ: median $M(Y) = \mu$ and the median absolute deviation $MAD(Y) \equiv M(|Y - M(Y)|) \simeq 0.6745\sigma$
- Kolmogorov-Smirnov tests: p-values by $10,000$ random samples generated under $N(\hat{\mu}, \hat{\sigma}^2)$

skewness test based on: $$\frac{[Q_{1-p}(Y) - M(Y)] - [M(Y) - Q_p(Y)]}{Q_{1-p}(Y) - Q_p(Y)},$$

where $Q_\alpha(Y)$ is the α-th percentile.

kurtosis test based on: $$\frac{[O_7(Y) - O_5(Y)] + [O_3(Y) - O_1(Y)]}{O_6(Y) - O_2(Y)},$$

where $O_\alpha(Y)$ is the α-th octile.

- **Figure 2a-d, US; Figure 3a-c, UK.**
  - Log normal distribution of equivalised consumption by cohort and time.
- Gibrat's law over the life-cycle for consumption rather than income?
  - Extend the Deaton-Paxson *JPE* result on the variances of log consumption over the life-cycle
  - There are many alternative regularity conditions that will yield a CLT, they all require a uniform asymptotic negligibility condition (relating to existence of moments) and a limit on the degree of dependence of observations over time such as alpha mixing.
- **Figure 4a-d**

Income dynamics (1)

General specification for income dynamics for consumer i of age a in time period t.

Write log income $\ln Y_{i,a,t}$ as:

$$y_{i,a,t} = B'_{i,a,t} f_i + Z'_{i,a,t}\varphi + y^P_{i,a,t} + y^T_{i,a,t} \tag{1}$$

- where y^P_{it} is a persistent process of income shocks which adds to the individual-specific trend (by age and time) $B'_{i,a,t} f_i$ and where y^P_{it} is a transitory shock represented by some low order MA process.
- Allow variances (or factor loadings) of y^P and y^T to vary with cohort, time,..
- For any cohort, an interesting possible specification for $B'_{i,t} f_i$ is

$$B'_{i,t} f_i = p_t f_{1i} + f_{0i} \tag{2}$$

- If $y_{i,t}^T$ is represented by a MA(q)

$$v_{it} = \sum_{j=0}^{q} \theta_j \varepsilon_{i,t-j} \text{ with } \theta_0 \equiv 1. \tag{3}$$

- and y_{it}^P by

$$y_{it}^P = \rho y_{it-1}^P + \zeta_{it}, \tag{4}$$

With $q = 1$, this implies a 'key' quasi-difference moment restriction

$$\mathrm{cov}(\Delta^\rho y_t, \Delta^\rho y_{t-2}) = \mathrm{var}(f_0)(1-\rho)^2 + \mathrm{var}(f_1)\Delta^\rho p_t \Delta^\rho p_{t-2} - \rho\theta_1 \mathrm{var}(\varepsilon_{t-2}) \tag{5}$$

where $\Delta^\rho = (1 - \rho L)$ is the quasi-difference operator.

- Note that for large $\rho = 1$ and small θ_1 this implies

$$\mathrm{cov}(\Delta y_t, \Delta y_{t-2}) \simeq \mathrm{var}(f_1)\Delta p_t \Delta p_{t-2}. \tag{6}$$

Idiosyncratic trends:

► The term $p_t f_{1i}$ could take a number of forms

(a) deterministic idiosyncratic trend : $p_t f_{1i} = r(t) f_{1i}$ where r is known, e.g. $r(t) = t$

(b) stochastic trend in 'ability prices' : $p_t = p_{t-1} + \xi_t$ with $E_{t-1}\xi_t = 0$

► Evidence points to some periods of time where each is of key importance:

- (a) early in working life (Solon et al.). Formally, this is a life-cycle effect.
- (b) during periods of technical change when skill prices are changing across the unobserved ability distribution. Early 1980s in the US and UK, for example. Formally, this is a calender time effect.

► I will come back to look at various sensitivity results for ρ and $p_t f_{1i} + f_{0i}$.

Income dynamics (2)

- For each household i, I first consider a simple permanent-transitory decomposition for log income:

$$y_{it} = Z'_{it}\varphi + y^P_{it} + y^T_{it} \tag{7}$$

- with

$$y^P_{it} = y^P_{it-1} + \zeta_{it} \tag{8}$$

- and transitory or mean-reverting component, $y^T_{it} = v_{i,t}$

$$v_{it} = \sum_{j=0}^{q} \theta_j \varepsilon_{i,t-j} \text{ with } \theta_0 \equiv 1. \tag{9}$$

- Implies a restrictive structure for the autocovariances of $\Delta y_{it} (= \zeta_{it} + \Delta v_{it})$, where $y_{it} = \log Y_{it} - Z'_{it}\varphi$.

Some (Simple) Empirics

- How well does it work?
- **Tables III a, b and c** present the autocovariance structure of the PSID and the BHPS (ECFP and JPID on my webpage).
- this latent factor structure aligns 'well' with the autocovariance structure of the PSID, the BHPS (UK), JPID(Japan) and the ECFP(Spain)
    - allows for general fixed effects and initial conditions.
    - regular deconvolution arguments lead to identification of variances and complete distributions, e.g. Bonhomme and Robin (2006)
    - the key idea is to allow the variances (or loadings) of the factors to vary nonparametrically with cohort, education and time: - the relative variance of these factors is a measure of persistence or durability of labour income shocks.

Evolution of the Consumption Distribution

- with Self-Insurance

► At time t each individual i maximises the conditional expectation of a time separable, differentiable utility function:

$$\max_C E_t \sum_{j=0}^{T-t} u\left(C_{i,t+j}, Z_{i,t+j}\right)$$

$Z_{i,t+j}$ incorporates taste shifters/non-separabilities and discount rate heterogeneity.

• We set the retirement age at L, assumed known and certain, and the end of the life-cycle at T. We assume that there is no uncertainty about the date of death.

• Individuals can self-insure using a simple credit market with access to a risk free bond with real return r_{t+j}. Consumption and income are linked through the intertemporal budget constraint

$$A_{i,t+j+1} = \left(1 + r_{t+j}\right)\left(A_{i,t+j} + Y_{i,t+j} - C_{i,t+j}\right) \text{ with } A_{i,T} = 0.$$

Consumption Dynamics (1)

- With self-insurance and CRRA preferences

$$u\left(C_{i,t+j}, Z_{i,t+j}\right) \equiv \frac{1}{(1+\delta)^j} \frac{C_{i,t+j}^{\beta} - 1}{\beta} e^{Z'_{i,t+j}\vartheta}$$

  - The first-order conditions become

$$C_{i,t-1}^{\beta-1} = \frac{1+r_{t-1}}{1+\delta} e^{\Delta Z'_{i,t}\vartheta_t} E_{t-1} C_{i,t}^{\beta-1}.$$

- Applying an exact Taylor series approximation (see BLP) to the Euler equation above gives

$$\Delta \log C_{i,t} \simeq \Delta Z'_{i,t}\vartheta'_t + \eta_{i,t} + \Omega_{i,t}$$

where $\vartheta'_t = (1-\beta)^{-1}\vartheta_t$, $\eta_{i,t}$ is a consumption shock with $E_{t-1}\eta_{i,t} = 0$, $\Omega_{i,t}$ captures any slope in the consumption path due to interest rates, impatience or precautionary savings and the error in the approximation is $O(E_t\eta_{i,t}^2)$.

Linking the Evolution of Consumption and Income Distributions

▶ We write any idiosyncratic component to the gradient to the consumption path as a vector of deterministic characteristics $\Gamma_{i,t}$ and a stochastic individual element $\xi_{i,t}$

$$\Delta \ln C_{i,t} \simeq \Gamma_{i,t} + \Delta Z'_{i,t}\vartheta'_t + \eta_{i,t} + \xi_{i,t}.$$

- For income we have

$$\Delta \ln Y_{i,t+k} = \zeta_{i,t+k} + \sum_{j=0}^{q} \theta_j \varepsilon_{i,t+k-j}.$$

- The intertemporal budget constraint is

$$\sum_{k=0}^{T-t} Q_{t+k} C_{i,t+k} = \sum_{k=0}^{L-t} Q_{t+k} Y_{i,t+k} + A_{i,t}$$

where T is death, L is retirement and Q_{t+k} is appropriate discount factor $\prod_{i=1}^{k}(1 + r_{t+i})$, $k = 1, ..., T - t$ (and $Q_t = 1$).

- Use exact Taylor series expansion, as developed in BLP and BPP.

▶ Applying the BPP approximation appropriately to each side

$$\sum_{k=0}^{T-t} \alpha_{t+k,T}^{\Omega-r}[\ln C_{i,t+k} - \ln Q_{t+k} - \ln \alpha_{t+k,T}^{\omega-r}] \simeq \pi_{i,t} \sum_{k=0}^{L-t} \alpha_{t+k,L}^{-r}[\ln Y_{i,t+k} - \ln Q_{t+k} - \ln \alpha_{t+k,L}^{-r}]$$
$$+ (1-\pi_{i,t}) \ln A_{i,t} - [(1-\pi_{i,t}) \ln(1-\pi_{i,t}) + \pi_{i,t} \mathrm{l}$$

- where $\pi_{i,t} = \frac{\sum_{k=0}^{L-t} Q_{t+k} Y_{i,t-k}}{\sum_{k=0}^{L-t} Q_{t+k} Y_{i,t-k} + A_{i,t}}$ is the share of future labor income in current human and financial wealth.

- Taking differences in expectations we have the consumption growth shock

$$\eta_{i,t} \simeq \pi_{i,t} \left[\zeta_{i,t} + \gamma_{t,L} \varepsilon_{i,t}\right]$$

where the error on the approximation is $O(\left[\zeta_{i,t} + \gamma_{t,L}\varepsilon_{i,t}\right]^2 + E_{t-1}\left[\zeta_{i,t} + \gamma_{t,L}\varepsilon_{i,t}\right]^2)$.

  - If $r_t = r$ is constant then $\gamma_{t,L}$ is the annuity expression

$$\gamma_{t,L} \simeq \frac{r}{1+r}[1 + \sum_{j=1}^{q} \theta_j/(1+r)^j].$$

So **a link between consumption and income dynamics** can be expressed, to order $\mathcal{O}(\|\nu_t\|^2)$, where $\nu_t = (\zeta_t, \varepsilon_t)'$

$$\Delta \ln C_{it} \cong \Gamma_{it} + \Delta Z'_{it}\varphi^c + \pi_{it}\zeta_{it} + \pi_{it}\gamma_{Lt}\varepsilon_{it} + \xi_{it}$$

- Γ_{it} - Impatience, precautionary savings, intertemporal substitution. For CRRA preferences Γ does not depend on C_{t-1}.
- $\Delta Z'_{it}\varphi^c$ - Deterministic preference shifts and labor supply non-separabilities
- $\pi_{it}\zeta_{it}$ - Impact of permanent income shocks - $1 - \pi_{it}$ reflects the degree to which 'permanent' shocks are insurable in a finite horizon model.
- $\pi_{it}\gamma_{Lt}\varepsilon_{it}$ - Impact of transitory income shocks, $\gamma_{Lt} < 1$ - an annuitisation factor
- ξ_{it} - Impact of shocks to higher income moments,etc

The π parameter

In this model, self-insurance is driven by the parameter π, which corresponds to the ratio of human capital wealth to total wealth (financial + human capital wealth)

$$\pi_{i,t} = \frac{\sum_{k=0}^{L-t} Q_{t+k} Y_{i,t-k}}{\sum_{k=0}^{L-t} Q_{t+k} Y_{i,t-k} + A_{i,t}}$$

- For given level of human capital wealth, past savings imply higher financial wealth today, and hence a lower value of π: Consumption responds less to income shocks (precautionary saving)
  - Individuals approaching retirement have a lower value of π
  - In the certainty-equivalence version of the PIH, $\pi \simeq 1$ and $\alpha \simeq 0$

When Does Consumption Inequality Measure Welfare Inequality?

Define $\widetilde{Y}_i$ as that certain present discounted value of lifetime income which would allow the individual to achieve the same expected utility. The consumption stream $\widetilde{\mathbf{C}}_i = \widetilde{\mathbf{C}}(EU_i)$ that would be chosen given $\widetilde{Y}_i$ satisfies

$$\sum_t u_t(\widetilde{C}_{it}) \equiv E(\sum_t u_t(C_{it})) = EU_i.$$

PROPOSITION 1 *Comparisons across individuals facing different income risk*: $C_{it} \geq C_{jt}$ implies $EU_i \geq EU_j$ whenever individuals i and j share the same year of birth if and only if $\mathbf{C}_i = \widetilde{\mathbf{C}}(EU_i)$ whatever the distribution of future income. This is so if and only if $u_t(C_{it}) = -\alpha_t \exp(-\gamma_t C_{it}) \qquad \alpha_t, \gamma_t > 0, t > 0.$

- This holds exactly iff CARA. The sufficiency part is a special case of a more general result that decreasing absolute risk aversion (DARA) implies $C_{i0} < \widetilde{C}_{i0}$, ie that there is excess precautionary saving if higher incomes decrease risk aversion.

Moral hazard, Limited enforcement

- Under some circumstances, it is possible to insure consumption fully against income shocks. In this case, $\pi = 0$
    - Theoretical problems: Moral hazard, Limited enforcement, etc.
    - Empirical problems: The hypothesis $\pi = 0$ is soundly rejected, refs.
- Introduce ‘partial insurance’ to capture the possibility of ‘excess insurance’ and also ‘excess sensitivity’.

Partial Insurance

- The stochastic Euler equation is consistent with many stochastic processes for consumption. It does not say anything about the variance of consumption.

- In the full information perfect market model with separable preferences the variance of consumption is zero. In comparison with the self-insurance model the intertemporal budget constraint based on a single asset is violated.

- Partial insurance allows some additional insurance. For example, Attanasio and Pavoni (2005) consider an economy with moral hazard and hidden asset accumulation - individuals now have hidden access to a simple credit market. They show that, depending on the cost of shirking and the persistence of the income shock, some partial insurance is possible. A linear insurance rule can be obtained as an 'exact' solution in a dynamic Mirrlees model with CRRA utility.

Consumption dynamics (2) - Partial Insurance

Need to generalise to account for additional 'insurance' mechanisms and excess sensitivity - introduce *transmission parameters* ϕ_{bt} and ψ_{bt}

$$\Delta \ln C_{it} \cong \Gamma_{it} + \Delta Z'_{it}\varphi^{c} + \xi_{it} + \phi_{bt}\zeta_{it} + \psi_{bt}\varepsilon_{it}$$

where b is the birth cohort for individual i.

- Partial insurance w.r.t. permanent shocks, $0 \leq 1 - \phi_{bt} \leq 1$

- Partial insurance w.r.t. transitory shocks, $0 \leq 1 - \psi_{bt} \leq 1$

- $1-\ \phi_{bt}$ and $1 - \psi_{bt}$ are the fractions insured and subsume π and γ from the self-insurance model

- This factor structure provides the key panel data moments that link the evolution of distribution of consumption to the evolution of labour income distribution.

A Factor Structure for Consumption and Income Dynamics

- We now have a factor structure provides the key panel data moments that link the evolution of distribution of consumption to the evolution of labour income distribution

$$\Delta \ln C_{it} \cong \Gamma_{it} + \Delta Z_{it}' \varphi^c + \phi_{bt} \zeta_{it} + \psi_{bt} \varepsilon_{it} + \xi_{it}$$

- It describes how consumption updates to income shocks
- It provides key panel data moments that link the evolution of distribution of consumption to the evolution of income
- We can compare it with results from a dynamic stochastic simulation of a Bewley economy and other common alternatives...

The key panel data moments

- For log income:

$$\operatorname{cov}(\Delta y_t, \Delta y_{t+s}) = \begin{cases} \operatorname{var}(\zeta_t) + \operatorname{var}(\Delta v_t) & \text{for } s = 0 \\ \operatorname{cov}(\Delta v_t, \Delta v_{t+s}) & \text{for } s \neq 0 \end{cases} \tag{10}$$

- Allowing for an MA(q) process, for example, adds $q-1$ extra parameter (the $q-1$ MA coefficients) but also $q-1$ extra moments, so that identification is unaffected.

- For log consumption:

$$\operatorname{cov}(\Delta c_t, \Delta c_{t+s}) = \phi_{b,t}^2 \operatorname{var}(\zeta_t) + \psi_{b,t}^2 \operatorname{var}(\varepsilon_t) + \operatorname{var}(\xi_t) \tag{11}$$

for $s = 0$ and zero otherwise.

- For the cross-moments:

$$\operatorname{cov}(\Delta c_t, \Delta y_{t+s}) = \begin{cases} \phi_{b,t} \operatorname{var}(\zeta_t) + \psi_{b,t} \operatorname{var}(\varepsilon_t) \\ \psi_{b,t} \operatorname{cov}(\varepsilon_t, \Delta v_{t+s}) \end{cases} \tag{12}$$

for $s = 0$, and $s > 0$ respectively.

• A simple summary the panel data moments:

$$
\begin{aligned}
\operatorname{var}(\Delta y_t) &= \operatorname{var}(\zeta_t) + \operatorname{var}(\Delta\varepsilon_t) \\
\operatorname{cov}(\Delta y_{t+1}, \Delta y_t) &= -\operatorname{var}(\varepsilon_t) \\
\operatorname{var}(\Delta c_t) &= \phi_t^2 \operatorname{var}(\zeta_t) + \psi_t^2 \operatorname{var}(\varepsilon_t) + \operatorname{var}(\xi_{\iota t}) + \operatorname{var}(u_{it}^c) \\
\operatorname{var}(\Delta c_t, \Delta c_{it+1}) &= -var(u_{it}^c) \\
\operatorname{cov}(\Delta c_t, \Delta y_t) &= \phi_t \operatorname{var}(\zeta_t) + \psi_t \operatorname{var}(\varepsilon_t) \\
\operatorname{cov}(\Delta c_t, \Delta y_{t+1}) &= -\psi_t \operatorname{var}(\varepsilon_t)
\end{aligned}
$$

• Under additional assumptions, Blundell and Preston (QJE, 1998) turn these into identifying moments for repeated cross-section data. I'll return to the evolution of cross-section moments.

• Also assess the validity of the approximation we simulate a stochastic economy....

More on Identification

The model can be identified with four years of data ($t+1, t, t-1, t-2$). Start with the simplest model with no measurement error, serially uncorrelated transitory component, and stationarity.

► The parameters to identify are: $\phi, \psi, \sigma_\xi^2, \sigma_\zeta^2$, and σ_ε^2.

- Standard results imply: $E\left(\Delta y_t\left(\Delta y_{t-1}+\Delta y_t+\Delta y_{t+1}\right)\right)=\sigma_\zeta^2$
- and also that: $E\left(\Delta y_t \Delta y_{t-1}\right)=E\left(\Delta y_{t+1} \Delta y_t\right)=-\sigma_\varepsilon^2$
- Identification of σ_ε^2 rests on the idea that income growth rates are autocorrelated due to mean reversion caused by the transitory component
- Identification of σ_ζ^2 rests on the idea that the variance of income growth ($E\left(\Delta y_t \Delta y_t\right)$), less the contribution of the mean reverting component ($E\left(\Delta y_t \Delta y_{t-1}\right)+E\left(\Delta y_t \Delta y_{t+1}\right)$), coincides with the permanent innovations.

► In general, if one has T years of data, only $T-3$ variances of the permanent shock can be identified, and only $T-2$ variances of the i.i.d. transitory shock can be identified.

Also prove that:

- $E\left(\Delta c_t\left(\Delta y_{t-1}+\Delta y_t+\Delta y_t\right)\right)/E\left(\Delta y_t\left(\Delta y_{t-1}+\Delta y_t+\Delta y_t\right)\right)=\phi$
- $E\left(\Delta c_t \Delta y_{t+1}\right)/E\left(\Delta y_t \Delta y_{t+1}\right)=\psi$
- $E\left(\Delta c_t\left(\Delta c_{t-1}+\Delta c_t+\Delta c_{t+1}\right)\right)-\frac{\left[E(\Delta c_t(\Delta y_{t-1}+\Delta y_t+\Delta y_{t+1}))\right]^2}{E(\Delta y_t(\Delta y_{t-1}+\Delta y_t+\Delta y_{t+1}))}+\frac{\left[E(\Delta c_t \Delta y_{t+1})\right]^2}{E(\Delta y_t \Delta y_{t+1})}=\sigma_{\xi}^2.$

Identification of ψ using uses the fact that income and lagged consumption may be correlated through the transitory component ($E\left(\Delta c_t \Delta y_{t+1}\right)=\psi \sigma_{\varepsilon}^2$). Scaling this by $E\left(\Delta y_t \Delta y_{t+1}\right)=\sigma_{\varepsilon}^2$ identifies the loading factor ψ.

- Note that there is a simple IV interpretation here: ψ is identified by a regression of Δc_t on Δy_t using Δy_{t+1} as an instrument.

A similar reasoning applies to ϕ where the current covariance between consumption and income growth ($E\left(\Delta c_t \Delta y_t\right)$), stripped of the contribution of the transitory component, reflects the arrival of permanent income shocks ($E\left(\Delta c_t\left(\Delta y_{t-1}+\Delta y_t+\Delta y_t\right)\right)=\phi\sigma_\zeta^2$). Scaling this by the variance of permanent income shock, identified by using income moments alone, identifies the loading factor ϕ.

- Note again a simple IV interpretation: ϕ is identified by a regression of Δc_t on Δy_t using $\left(\Delta y_{t-1}+\Delta y_t+\Delta y_{t+1}\right)$ as an instrument.
- The variance of the component σ_ξ^2 is identified using a residual variability idea: the variance of consumption growth, stripped of the contribution of permanent and transitory income shocks, reflects heterogeneity in the consumption gradient.

Measurement error in consumption.

$$c^*_{i,t} = c_{i,t} + u^c_{i,t}$$

where c^* denote measured consumption, c is true consumption, and u^c the measurement error.

- Measurement error in consumption induces serial correlation in consumption growth. Because consumption is a martingale with drift in the absence of measurement error, the variance of measurement error can be recovered using

$$E\left(\Delta c^*_t \Delta c^*_{t-1}\right) = E\left(\Delta c^*_t \Delta c^*_{t+1}\right) = -\sigma^2_{u^c}$$

- The other parameters of interest remain identified. One obvious reason for the presence of measurement error in consumption is our imputation procedure - we expect the measurement error to be non-stationary (which we account for in estimation).

Measurement error in income

$$y^*_{i,t} = y_{i,t} + u^y_{i,t}$$

• Can show ϕ and $\sigma^2_{u^c}$ are still identified. However, σ^2_{ε} and $\sigma^2_{u^y}$ cannot be separated, and ψ (as well as σ^2_{ξ}) thus remains unidentified.

• It is possible however to put a lower bound on ψ as the estimate is downward biased. For the PSID, a back-of-the-envelope calculation shows that the variance of measurement error in earnings accounts for approximately 30 percent of the variance of the overall transitory component of earnings.

• Given that our estimate of ψ is close to zero in most cases, an adjustment using this inflation factor would make little difference empirically. Using a similar reasoning, one can argue that we have an upper bound for σ^2_{ξ}. The bias, however, is likely negligible.

Non-stationarity.

Allowing for non-stationarity and with T years of data

$$E\left(\Delta y_s^*\left(\Delta y_{s-1}^*+\Delta y_s^*+\Delta y_{s+1}^*\right)\right)=\sigma_{\zeta,s}^2$$

for $s=3,4,...,T-1$. The variance of the transitory shock can be identified using:

$$-E\left(\Delta y_s^*\Delta y_{s+1}^*\right)=\sigma_{\varepsilon,s}^2$$

for $s=2,3,...,T-1$. With an MA(1) process for the transitory component:

$$E\left(\Delta y_s^*\left(\Delta y_{s-2}^*+\Delta y_{s-1}^*+\Delta y_s^*+\Delta y_{s+1}^*+\Delta y_{s+2}^*\right)\right)=\sigma_{\zeta,s}^2$$

for $s=4,5...,T-2$, and (assuming θ is already identified)

$$-E\left(\Delta y_s^*\Delta y_{s+2}^*\right)=\theta\sigma_{\varepsilon,s}^2$$

for $s=2,3,...,T-2$.

- The other parameters of interest ($\sigma_{u^c}^2, \phi, \psi$,$\sigma_\xi^2$) can be identified.

Time-varying insurance parameters

.

$$\Delta c_s = \xi_s + \phi_s \zeta_s + \psi_s \varepsilon_s + \Delta u_s^c$$

- which would be identified by the moment conditions:

$$\frac{E(\Delta c_s^* \Delta y_{s+1}^*)}{E\left(\Delta y_s^* \Delta y_{s+1}^*\right)} = \psi_s$$
$$\frac{E(\Delta c_s^* (\Delta y_{s-1}^* + \Delta y_s^* + \Delta y_{s+1}^*))}{E\left(\Delta y_s^* \left(\Delta y_{s-1}^* + \Delta y_s^* + \Delta y_{s+1}^*\right)\right)} = \phi_s$$

for all $s = 2, 3, ..., T-1$ and $s = 3, 4, ..., T-2$ respectively.

- These are the moment conditions that we use when we allow the insurance parameters to vary over time.

The US PSID/CEX Data

◀ PSID 1968-1996: (main sample 1978-1992)

- Construct all the possible panels of 5 $\leq$ length $\leq$ 15 years
- Sample selection: male head aged 30-62, no SEO/Latino subsamples

◀ CEX 1980-1998: (main sample 1980-1992)

- Focus on 5-quarters respondents only (annual expenditure measures)
- Sample selection similar to the PSID

◀ A comparison of both data sources is in Blundell, Pistaferri and Preston (2004)

- Note also the source for the UK BHPS, Spanish ECFP and Japanese panel

Linking consumption data in the CEX with the Income panel data in the PSID

- Food consumption, income and total expenditure in CEX, but a repeated cross-section

- Food consumption and income in the PSID panel.
  - ▷ Plus lots of demographic and other matching information in each year.

- Inverse structural demand equation acts as an 'imputation' equation - (**Table II in BPP**).

- Implications for consumption and income inequality - **Figure 5**

- Covariance structure of consumption and income **- Table V in BPP**

Partial Insurance and the other 'structural' parameters

- "excess smoothness" or "excess insurance" relative to self-insurance

Table VI:

- College-no college comparison
- Younger versus older cohorts

Figures 6,7: show implications for variances of permanent and transitory shocks

- Within cohort and education analysis changes the balance between the distribution of permanent and transitory shocks but not the value of the transmission parameters.

    - Strongly reject constancy of ϕ and ψ when food in PSID is used (Table AII)

Partial Insurance: Family Transfers and Taxes

Table VII:

● Tax system and transfers provide some insurance to permanent shocks

▷ food stamps for low income households studied in Blundell and Pistaferri (2003), ‘Income volatility and household consumption: The impact of food assistance programs’, special conference issue of JHR,

▷ also contains the Meyer and Sullivan paper, ‘Measuring the Well-Being of the Poor Using Income and Consumption’

▷ little impact of measured family transfers

Partial Insurance: Wealth

- Excess sensitivity among low wealth households: select (30%) initial low wealth.

Table VIII

- Excess sensitivity among low wealth households
    - Excess sensitivity among low wealth households - use of durables among low wealth households? - more later

Summary so far....

◀ The aim: to analyse the transmission from income to consumption inequality

◀ Specifically to examine the disjuncture in the evolution of income and consumption inequality in the US & UK in the 1980s - argue that a key driving force is the nature and the durability of shocks to labour market earnings

- a dramatic change in the mix of permanent and transitory income shocks over this period - revisionists?
- the growth in the persistent factor during the early 1980s inequality growth episode carries through into consumption

◀ But the transmission parameter is too small relative to the standard incomplete markets model

- about 30% of permanent shocks are insured (but not for the low wealth).

Further Issues

- Alternative income dynamics: robustness?
- What if we ignore the distinction between permanent and transitory shocks?
- What if we use food consumption data alone?
- Is there evidence of anticipation?

The Permanent-Transitory Distinction

◀ Suppose we ignore the distinction between permanent and transitory shocks

- The partial insurance coefficient is now a weighted average of the coefficients of partial insurance ϕ and ψ, with weights given by the importance of the variance of permanent (transitory) shocks

- Thus, one will have the impression that insurance is growing. But is the relative importance of more insurable shocks that is growing.

Food Data Alone

◀ Suppose we replicate the same analysis using food data

- This means there's no need to impute

- The transmission coefficients are now the product of two things: partial insurance and the budget elasticity of food consumption

- In the data, these coefficients fall over time, i.e., one finds evidence that insurance has increased

- But this assumes that the budget elasticity of food consumption is constant over time

- In the data, this elasticity falls over time Thus, what is a decline in the relative importance of food in overall non-durable consumption is interpreted as an increase in the insurance of consumption with respect to income shocks

Anticipation

Test $cov(\Delta y_{t+1}, \Delta c_t) = 0$ for all t, p-value 0.3305

Test cov(Δy_{t+2}, Δc_t) = 0 for all t, p-value 0.6058

Test cov(Δy_{t+3}, Δc_t) = 0 for all t, p-value 0.8247

Test cov(Δy_{t+4},Δc_t) = 0 for all t, p-value 0.7752

◀ We find little evidence of anticipation.

◀ This 'suggests' the shocks that were experienced in the 1980s were largely unanticipated.

◀ These were largely changes in the returns to skills, shifts in government transfers and the shift of insurance from firms to workers.

What next?

◀ Robustness to assumptions about the nature of the economy and the nature of the shocks

- ▶ Simulation studies for panel data and cross-section distributions under alternative assumptions
  - Credit market and insurance assumptions
  - Persistence of ‘shocks’ and advance information

◀ Additional ‘Insurance’ Mechanisms?

- ▶ Individual and family labor supply
- ▶ Durable replacement

Alternative Representations

- The complete markets, PIH and autarky cases
- A Bewley economy
  - approximation on the distribution of π and borrowing constraints
- A simple partial insurance economy
  - all transitory shocks insurable and a component of permanent shocks
- A private information economy
  - with moral hazard and hidden asset accumulation - linear insurance rule as a

solution in a Mirrlees model with CRRA utility.

- Advance information
  - a proportion of the shocks are known in advance to the consumer
  - know returns from human capital correlated with initial conditions

A Bewley economy

► Life-cycle version of the standard incomplete markets model e.g. Huggett (1993).

• Markets are incomplete: the only asset available is a single risk-free bond.

• Households have time-separable expected CRRA utility

$$E_0 \sum_{t=1}^{T} \beta^{t-1} m_t u(C_{it})$$

• Households enter the labor market at age 25, retire at age 60 and die at age 100.

• Assume survival rate $m_t = 1$ for the first T^{work} periods, so that there is no chance of dying before retirement.

• Discount factor: $.964$ with interest rate to match an aggregate wealth-income ratio of 3.5.

► **Income process**:

- Stochastic after-tax income, Y_{it}: deterministic experience profile, a permanent and transitory component; initial permanent shock is drawn from normal distribution.
- deterministic age profile for income from PSID data, peaks after 21 years at twice the initial value and then declines to about 80% of peak.
- variance of permanent shocks 0:02; variance of transitory shocks 0:05; as in BPP.
- The initial variance is set at 0:15 to match the dispersion at age 25.
- Households begin their life with initial wealth A_{i0}, face a lower bound on assets $\underline{A}$.
- Treat income Y_{it} as net household income after all transfers and taxes, also consider taxes on labor income through a non-linear tax rule $\tau(Y_{it})$ reflecting the redistribution in the US tax system.

- **Results**

  - Based on simulating, from the invariant distribution of the economy, an artificial panel of 50,000 households for 71 periods, i.e. a life-cycle

Go to tables

  - Also simulate a 'repeated cross' section data and use cross-section moments alone - return to this.

- **Table IX baseline**
- **Table X sensitivity to EIS, etc**
- **Table XIa,b: advance information**
- **Table XII: persistence of shocks**

- **Advance information I**
  - a proportion of the shocks are known in advance to the consumer
  - the permanent change in income at time t consists of two orthogonal components, one that becomes known to the agent at time t, the other is in the agent's information set already at time $t-1$.
- **Advance information II:**
  - the income process in includes heterogeneous slopes in individual income profiles:

$$y_{it} = f_{1i}t + y_{it}^{P} + \varepsilon_{it}$$

with $E(f_{1i}) = 0$, in the cross-section and $var(f_{1i}) = \sigma_{\beta}$, assume that f_{1i} is learned by the agents at age zero.

Additional ‘Insurance’ Mechanisms

- Family Labour Supply: Wages ▶ earnings ▶ joint earnings ▶ income ...
  - Stephens; Heathcote, Storesletten and Violante; Attanasio, Low and Sanchez-Marcos
- Redistributive mechanisms: social insurance, transfers, progressive taxation
  - Gruber; Gruber and Yelowitz; Blundell and Pistaferri; Kniesner and Ziliak
- Family and interpersonal networks
  - Kotlikoff and Spivak; Attanasio and Rios-Rull
- Durable replacement
  - Browning and Crossley

Family Labour Supply

▶ Total income Y_t is the sum of two sources, Y_{1t} and $Y_{2t} \equiv W_t h_t$

- Assume the labour supplied by the primary earner to be fixed. Income processes

$$\Delta \ln Y_{1t} = \gamma_{1t} + \Delta u_{1t} + v_{1t}$$

$$\Delta \ln W_t = \gamma_{2t} + \Delta u_{2t} + v_{2t}$$

- Household decisions to be taken to maximise a household utility function

$$\sum_k (1+\delta)^{-k} [U(C_{t+k}) - V(h_{t+k})].$$

$$\begin{aligned} \Delta \ln C_{t+k} &\simeq \sigma_{t+k} \Delta \ln \lambda_{t+k} \\ \Delta \ln h_{t+k} &\simeq -\rho_{t+k} [\Delta \ln \lambda_{t+k} + \Delta \ln W_{t+k}] \end{aligned}$$

with $\sigma_t \equiv U_t'/C_t U_t'' < 0$, $\rho_t \equiv -V_t'/h_t V_t'' > 0$.

- These imply second order panel data moments for $\ln C$, $\ln Y_1$, $\ln Y_2$ and $\ln W$.

▶ **The key panel data moments** become:

$$\begin{aligned}
Var(\Delta c_t) &\simeq \beta^2\sigma^2 s^2 Var(v_{1t}) + \beta^2\sigma^2(1-\rho)^2(1-s)^2 Var(v_{2t}) \\
&\quad +2\beta^2\sigma^2(1-\rho)s(1-s)Cov(v_{1t}, v_{2t}) \\
Var(\Delta y_{1t}) &\simeq Var(v_{1t}) + \Delta Var(u_{1t}) \\
Var(\Delta y_{2t}) &\simeq (1-\psi)^2 Var(u_{2t}) - \beta^2\rho^2 s^2 Var(v_{1t}) \\
&\quad +\beta^2\sigma^2(1-\rho)^2 Var(v_{2t}) - 2\beta^2\sigma(1-\rho)sCov(v_{1t}, v_{2t}) \\
Var(\Delta w_t) &\simeq Var(v_{2t}) + \Delta Var(u_{2t})
\end{aligned}$$

where

- $\beta = 1/(\sigma + \rho(1-s))$.
- s_t is the ratio of the mean value of the primary earner's earnings to that of the

household $\overline{Y}_{1t}/\overline{Y}_t$

► These moments are sufficient to identify permanent and transitory shock distribution, and their evolution over time, for $\ln Y_1$ and $\ln W$.

• When the labour supply elasticity $\rho > 0$ then the secondary worker provides insurance for shocks to Y_1

• **Figure 8**: shows implications for the variance of transitory shocks to household income and reconciles the Gottshalk and Moffitt results

• Impact of labour supply as a smoothing mechanism? **Table XIV**

See also *Attanasio, Berloffa, Blundell and Preston* (2002, EJ), 'From Earnings Inequality to Consumption Inequality', *Attanasio, Sanchez-Marcos and Low* (2005, JEEA), 'Female labor Supply as an Insurance Against Idiosyncratic Risk', and *Heathcote, Storesletten and Violante* (2006), 'Consumption and Labour Supply with Partial Insurance'

Partial Insurance: Durables

- We have seen excess sensitivity among low wealth households: select (30%) initial low wealth.

    also consider

- Impact of durable purchases as a smoothing mechanism?

Table XIV

- Excess sensitivity among low wealth households
- For poor households at least - absence of simple credit market
    - Excess sensitivity among low wealth households - even more impressive use of durables among low wealth households: - Browning, and Crossley (2003)

What about the evolution of Cross-section Distributions?

Assuming the cross-sectional covariances of the shocks with previous periods' incomes to be zero, then

$$\begin{aligned}
\Delta \mathrm{Var}(\ln y_t) &= \mathrm{Var}(v_t) + \Delta \mathrm{Var}(u_t) \\
\Delta \mathrm{Var}(\ln c_t) &= (\bar{\pi}_t{}^2 + \mathrm{Var}(\pi_t))\mathrm{Var}(v_t) + (\bar{\pi}_t{}^2 + \mathrm{Var}(\pi_t))\alpha_t^2 \mathrm{Var}(u_t) \\
&\quad + \mathrm{Var}(\pi_t)\Omega_t^2 \ + \ \mathcal{O}(E_{t-1}\|\nu_{it}\|^3) \\
\Delta \mathrm{Cov}(\ln c_t, \ln y_t) &= \bar{\pi}_t \mathrm{Var}(v_t) + \Delta[\bar{\pi}_t \alpha_t \mathrm{Var}(u_t)] \\
&\quad + \mathcal{O}(E_{t-1}\|\nu_{it}\|^3).
\end{aligned} \tag{13}$$

- Can identify variances of shocks and π
- **Figures 9, 10** show similar structure to US distributions from PSID.
- How well does this work? Back to simulated economy - calibrated to UK, **BLP**.

Simulation Experiments

- As before one aim of the Monte Carlo is to explore the accuracy with which the variances can be estimated despite the approximations. In particular, estimates of the permanent variance and of changes in the transitory variance.
- In the base case the subjective discount rate $\delta = 0.02$, also allow δ to take values 0.04 and 0.01. Also a mixed population with half at 0.02 and a quarter each at 0.04 and 0.01.
- In such cases the permanent variance follows a two-state, first-order Markov process with the transition probability between alternative variances, $\sigma^2_{\zeta,L}$ and $\sigma^2_{\zeta,H}$
- For each experiment, BLP simulate consumption, earnings and asset paths for 50,000 individuals. Obtain estimates of the variance for each period from random cross sectional samples of 2000 individuals for each of 20 periods: **Figure 11**

Idiosyncratic Consumption Trends:

Heterogeneous consumption trends Γ_{it}

$$\Delta \ln c_{it} = \varepsilon_{it} + \Gamma_{it} \; + \; \mathcal{O}(E_{t-1}{\varepsilon_{it}}^2)$$

the evolution of variances are modified to give:

$$\begin{aligned}
\Delta \mathrm{Var}\,(\ln y_t) &\simeq \mathrm{Var}(v_t) + \Delta \mathrm{Var}(u_t) \\
\Delta \mathrm{Cov}(\ln c_t, \ln y_t) &\simeq \bar{\pi}_t \,\mathrm{Var}(v_t) + \mathrm{Cov}(y_{t-1}, \Gamma_t) \\
\Delta \mathrm{Var}(\ln c_t) &\simeq \bar{\pi}_t^2 \,\mathrm{Var}(v_t) + 2\mathrm{Cov}(c_{t-1}, \Gamma_t)
\end{aligned}$$

- The evolution of $\mathrm{Var}(\ln c_t)$ is no longer usable since $\mathrm{Cov}(c_{t-1}, \Gamma_t) \neq 0$ for some t.
- The evolution of the cross-section variability in log consumption no longer reflects only the permanent component and so it cannot be used for identifying the variance of the permanent shock. **Figure 12**

Idiosyncratic Income Trends:

The equations for the evolution of the variances become:

$$\begin{aligned}
\Delta \mathrm{Var}(\ln y_t) &\simeq \mathrm{Var}(v_t) + \Delta \mathrm{Var}(u_t) + 2\mathrm{Cov}(y_{t-1}, \eta_t) \\
\Delta \mathrm{Cov}(\ln c_t, \ln y_t) &\simeq \bar{\pi}_t \, \mathrm{Var}(v_t) + \mathrm{Cov}(c_{t-1}, \eta_t) \\
\Delta \mathrm{Var}(\ln c_t) &\simeq \bar{\pi}_t{}^2 \, \mathrm{Var}(v_t)
\end{aligned}$$

where η reflects the idiosyncratic trend

- The evolution of the variance of income is no longer informative about uncertainty.
- The evolution of $\mathrm{Var}\,(\ln c_t)$ can be used to identify the variance of permanent shocks
- The evolution of the transitory variance cannot be identified
- The covariance term is useful only if the levels of consumption are uncorrelated with the income trend, which is unlikely. **Figure 13**

Simulation Experiments with Cross-section Distributions

- The aim is to show the accuracy with which the variances can be estimated despite these approximations. For example, the simulation model itself does allow fully for heterogeneity in π_t. In particular, the accuracy of estimates of the permanent variance and of changes in the transitory variance.
- In the base case the subjective discount rate $\delta = 0.02$, also allow δ to take values 0.04 and 0.01. Also a mixed population with half at 0.02 and a quarter each at 0.04 and 0.01.
- In such cases the permanent variance follows a two-state, first-order Markov process with the transition probability between alternative variances, $\sigma^2_{\zeta,L}$ and $\sigma^2_{\zeta,H}$
- For each experiment, BLP simulate consumption, earnings and asset paths for 50,000 individuals. Estimates from random cross section samples of 2000.

The variance of permanent shocks

- Transitory shocks are assumed to be $i.i.d.$ within period with variance growing at a deterministic rate.
- The permanent shocks are subject to stochastic volatility.
- The permanent variance as following a two-state, first-order Markov process with the transition probability between alternative variances, $\sigma^2_{v,L}$ and $\sigma^2_{v,H}$, given by β.

$$\begin{array}{c|cc|} & \sigma^2_{v,L} & \sigma^2_{v,H} \\ \hline \sigma^2_{v,L} & 1-\beta & \beta \\ \sigma^2_{v,H} & \beta & 1-\beta \\ \hline \end{array} \tag{14}$$

- Consumers believe that the permanent variance has an ex-ante probability β of changing in each t. In the simulations, the variance actually switches only once and this happens in period S, which we assume is common across all individuals.

Figure 5.

Summary

◀ The aim: to analyse the distributional dynamics from income to earnings to consumption inequality

◀ A specific case was the disjuncture in the evolution of income and consumption inequality in the US & UK in the 1980s

- argue that a key driving force is the nature and the durability of shocks to labour market earnings
- a dramatic change in the mix of permanent and transitory income shocks over this period - revisionists?
- the growth in the persistent factor during the early 1980s inequality growth episode carries through into consumption

► But the transmission parameter is too small relative to the standard incomplete markets model

- except for low education and low wealth families – 'partial insurance'.
- about 30% of permanent shocks are insured (but not for the poor or the low educated). An important insurance role is played by the tax system and welfare state (disability insurance, social security, food stamps, etc.).

◄ Transmission parameter approach is 'robust' but insurance interpretation sensitive to assumed/estimated persistence in the income series.

◄ Importance of low wealth and young adults ($<$30)

◄ Found family labour supply acts as insurance.

◄ Durable purchases as insurance to transitory shocks for lower wealth groups.

What of future research?

◀ Within household insurance: Heathcote, Storesletten and Violante (2006), Lise and Seitz (2005)

◀ Differential persistence across the distribution: optimal welfare results for low wealth/low human capital groups: optimal earned income tax-credits.

◀ Understanding the mechanism and market incentives for excess insurance - Krueger and Perri (2006) and Attanasio and Pavoni (2006).

◀ Advance information and/or predictable life-cycle income trends - Cuhna, Heckman and Navarro (2005).

◀ Alternative panel data income processes e.g. Guvenen (2006), Solon (2006), etc.

◀ The specific use of credit and durables - Browning and Crossley (2007)

◀ The role of housing – see recent disjuncture of the covariance series.....

THE END

What next?

◄ Robustness to relaxing assumptions about the nature of the economy and the nature of the shocks

◄ A role for durables?

◄ A role for family labour supply?

◄ Robustness to relaxing assumptions about the nature of the economy and the nature of the shocks

◄ Advance information and/or predictable life-cycle income trends - Cuhna, Heckman and Navarro (2005).

◄ Differential persistence across the distribution: optimal welfare results for low wealth/low human capital groups: optimal earned income tax-credits.

◄ Understanding the mechanism and market incentives for excess insurance -

Krueger and Perri (2006) and Attanasio and Pavoni (2006).

Information and the income process

It may be that the consumer cannot separately identify transitory ε_{it} from permanent ζ_{it} income shocks. For a consumer who simply observed the income innovation ϵ_{it} in $y_{it} = y_{i,t-1} + \epsilon_{it} - \theta_t \epsilon_{i,t-1}$ we have consumption innovation:

$$\eta_{it} = \rho_t (1 - \theta_{t+1}) \epsilon_{it} + \frac{r}{1+r} \theta_{t+1} \epsilon_{it} \tag{15}$$

where $\rho_t = 1 - (1+r)^{-(R-t+1)}$. The evolution of θ_t is directly related to the evolution of the variances of the transitory and permanent innovations to income.

- The permanent effects component in this decomposition can be thought of as capturing news about both current and *past* permanent effects since

$$E(\sum_{j=0} \zeta_{i,t-j} | \epsilon_{it}, \epsilon_{i,t-1}, \ldots) - E(\sum_{j=0} \zeta_{i,t-j} | \epsilon_{i,t-1}, \ldots) = (1 - \theta_{t+1}) \varepsilon_{it}.$$

  - This represents the best prediction of the permanent/ transitory split

Alternative Representations of the Economy

- A simple partial insurance economy
  - all transitory shocks insurable and a component of permanent shocks uninsurable
- A private information economy
  - with moral hazard and hidden asset accumulation - linear insurance rule as a solution in a Mirrlees model with CRRA utility.
- Advance information
  - a proportion of the shocks are known in advance to the consumer
  - know returns from human capital correlated with initial conditions

Linking the Dynamic Evolution of Income and Consumption Distributions

We begin by calculating the error in approximating the Euler equation.

$$E_t U'(c_{it+1}) = U'(c_{it}) \left(\frac{1+\delta}{1+r}\right) = U'(c_{it}\mathrm{e}^{\Gamma_{it+1}}) \tag{16}$$

for some Γ_{it+1}.

By exact Taylor expansion of period $t+1$ marginal utility in $\ln c_{it+1}$ around $\ln c_{it}+\Gamma_{it+1}$, there exists a $\tilde{c}$ between $c_{it}\mathrm{e}^{\Gamma_{it+1}}$ and c_{it+1} such that

$$\begin{aligned} U'(c_{it+1}) \;=\; & U'(c_{it}\mathrm{e}^{\Gamma_{it+1}}) \Bigg[1 + \frac{1}{\gamma(c_{it}\mathrm{e}^{\Gamma_{it+1}})}[\Delta \ln c_{it+1} - \Gamma_{it+1}] \\ & + \frac{1}{2}\beta(\tilde{c}, c_{it}\mathrm{e}^{\Gamma_{it+1}})[\Delta \ln c_{it+1} - \Gamma_{it+1}]^2\Bigg] \end{aligned} \tag{17}$$

where $\gamma(c) \equiv U'(c)/cU''(c) < 0$ and $\beta(\tilde{c}, c) \equiv \left[\tilde{c}^2 U'''(\tilde{c}) + \tilde{c}U''(\tilde{c})\right]/U'(c)$ and thus

$$\Delta \ln c_{it+1} = \Gamma_{it+1} - \frac{\gamma(c_{it}\mathrm{e}^{\Gamma_{it+1}})}{2} E_t\left\{\beta(\tilde{c}, c_{it}\mathrm{e}^{\Gamma_{it+1}})[\Delta \ln c_{it+1}\mathrm{e}^{\Gamma_{it+1}}]^2\right\} + \varepsilon_{it+1} \tag{18}$$

where the consumption innovation ε_{it+1} satisfies $E_t \varepsilon_{it+1} = 0$. As $E_t \varepsilon_{it+1}^2 \to 0$, $\beta(\tilde{c}, c_{it} \mathrm{e}^{\Gamma_{it+1}})$ tends to a constant and therefore by Slutsky's theorem

$$\Delta \ln c_{it+1} = \varepsilon_{it+1} + \Gamma_{it+1} \; + \; \mathcal{O}(E_t |\varepsilon_{it+1}|^2). \tag{19}$$

If preferences are CRRA then Γ_{it+1} does not depend on c_{it} and is common to all households, say Γ_{t+1}. The log of consumption therefore follows a martingale process with common drift

$$\Delta \ln c_{it+1} = \varepsilon_{it+1} + \Gamma_{t+1} \; + \; \mathcal{O}(E_t |\varepsilon_{it+1}|^2). \tag{20}$$

where ε_{it} is an innovation term; Γ_t is the common anticipated gradient to the consumption path, reflecting precautionary saving and intertemporal substitution, and $\mathcal{O}(x)$ denotes a term with the property that there exists a $K < \infty$ such that

$$|\mathcal{O}(x)| < K \, |x| \, .$$

considering cross-sectional variation in consumption,

$$\Delta \mathrm{Var}(\ln c_t) = \mathrm{Var}(\varepsilon_t) \; + \; \mathcal{O}(E_{t-1} \left|\varepsilon_{it}\right|^3)$$

The process for income is written

$$\Delta \ln y_{it} = \eta_t + \omega_t + \Delta u_{it} + v_{it}. \tag{21}$$

η_t is a deterministic trend and ω_t a stochastic term, both common to the members of the cohort, while v_{it} is a permanent idiosyncratic shock

$$\varepsilon_{it} \;=\; \pi_{it}(v_{it} + \alpha_t u_{it}) \; + \; \pi_{it}\Omega_t \; + \; \mathcal{O}_p\left(E_{t-1}\left\|\boldsymbol{\nu}_{it}^{R-1}\right\|^2\right)$$

and therefore

$$\Delta \ln c_{it} = \Gamma_t \; + \; \pi_{it}(v_{it} + \alpha_t u_{it}) \; + \; \pi_{it}\Omega_t \; + \; \mathcal{O}_p\left(E_{t-1}\left\|\boldsymbol{\nu}_{it}^{R-1}\right\|^2\right). \tag{22}$$

We assume the idiosyncratic shocks u_{it} and v_{it} are orthogonal and unpredictable given prior information so that

$$E\left(u_{it}|v_{it}, \boldsymbol{\nu}_{i1}^{t-1}, Y_{i0}\right) = E\left(v_{it}|u_{it}, \boldsymbol{\nu}_{i1}^{t-1}, Y_{i0}\right) \;=\; 0.$$

This is a popular specification compatible with an MA(1) process for idiosyncratic changes in log income. We make no assumptions about the time series properties of the common shocks ω_t.

We assume that the variances of the shocks v_{it} and u_{it} are the same in any period for all individuals in any cohort but allow that these variances are not constant over time and indeed can evolve stochastically. Define $\mathrm{Var}(u_t)$ to be the cross-section variance of transitory shocks in period t for a particular cohort and $\mathrm{Var}(v_t)$ to be the corresponding variance of permanent shocks. These are the idiosyncratic components of permanent and temporary risk facing individuals.

A More general model — Suppose consumption growth is now given by

$$\Delta c_s^* = \xi_s + \phi_0 \zeta_s + \phi_1 \zeta_{s-1} + \psi_0 \varepsilon_s + \psi_1 \varepsilon_{s-1} + \Delta u_s^c$$

while income growth is still:

$$\Delta y_s^* = \zeta_s + \varepsilon_s - \varepsilon_{s-1}$$

In this case, we assume consumption growth depends on current and lagged income shocks.

The parameters to identify (in the stationary case for simplicity) are $\phi_0, \phi_1, \psi_0, \psi_1, \sigma_\xi^2, \sigma_{u^c}^2, \sigma_\zeta^2$, and σ_ε^2. The variances of the income shocks are still identified by:

$$E\left(\Delta y_t^* \left(\Delta y_{t-1}^* + \Delta y_t^* + \Delta y_{t+1}^*\right)\right) = \sigma_\zeta^2$$

and:

$$E\left(\Delta y_t^* \Delta y_{t-1}^*\right) = E\left(\Delta y_{t+1}^* \Delta y_t^*\right) = -\sigma_\varepsilon^2$$

◀ However, only ψ_0 can be identified, using

$$\frac{E(\Delta c_t^* \Delta y_{t+1}^*)}{E\left(\Delta y_t^* \Delta y_{t+1}^*\right)} = \psi_0$$

while all the others remain not identified in the absence of further restrictions.

◀ The expression we used to identify ϕ in the baseline scenario,

$$\frac{E(\Delta c_t^* (\Delta y_{t-1}^* + \Delta y_t^* + \Delta y_{t+1}^*))}{E\left(\Delta y_t^* \left(\Delta y_{t-1}^* + \Delta y_t^* + \Delta y_{t+1}^*\right)\right)}$$

now identifies the sum $(\phi_0 + \phi_1)$.

- Increasing the number of lags of income shocks in the consumption income growth equation has no effects: ψ_0 is still identified, while only the sum of the ϕ parameters is identified.